Indian J Phys. **75B** (2), 97–104 (2001)

I J P B
— an international journal

Study of propagation of a Gaussian laser beam in parabolic index optical fiber having thermal nonlinearity

R K Khanna*
Plasma & Microwave Laboratory, Department of Physics, Government College, Ajmer-305 001, Rajasthan, India
and
A K Nagar
Department of Physics, Government Dungar College, Bikaner-334 001, Rajasthan, India
E-mail : rajendra khanna@yahoo com

Received 4 September 2000, accepted 22 January 2001

Abstract Steady state and time dependent analysis of propagation of a Gaussian laser beam in a parabolic refractive index profile silica dielectric fiber has been presented The effect of thermal nonlinearity, which acts as perturbation on the geometrically built-in radial inhomogeneity of refractive index, has been studied A paraxial ray approach has been adopted following Akhmanov and Sodha using the WKB approximation The dimensionless beamwidth parameter has been calculated for different axial points and the self-focusing and defocusing properties of such a medium have been studied for different values of related parameters If a positive value of dielectric constant is considered, focusing takes place. For the negative value, defocusing of the beam is seen A comparison of the steady state and time-dependent theories is made. It has been shown that the consideration of time factor gives new results to the propagation characteristics of the laser beam. In the intensity peaks due to self- focusing, a shift is noticed The critical power for self-focusing for the same radial distance is found to be different for the two theories. The propagation characteristics of the beam are found to be very sensitive with the effective value of the waveguide delta parameter. Results show a good agreement with the available experimental data and can be utilised in constructing new kind of thermal fiber devices.

Keywords Gaussian laser beam, propagation characteristics, thermal nonlinearity

PACS Nos. 42.65 Jx, 42 81 Ht, 42 65 Vh

1. Introduction

The phenomena of self-focusing has, for long, been employed for dispersionless propagation of laser beams in optical fibers and other nonlinear media [1,2]. Chiao *et al* demonstrated trapping behaviour of beam in a dielectric cell experimentally [3,4]. In early seventies, Hasegawa and Tappert [5] were successful in explaining the dispersionless propagation of laser pulse in silica dielectric fibers by utilising nonlinearity of the medium. Today, fiber optics technology is fast changing the communications senario of the world. Still, one has to go a long way in achieving the goal of dispersionless transmission over long distances. Propagation of trapped pulses in optical fibers, as a result of self-focusing, is of current interest because of its potential applications in optical switching, optical limiting, optical interconnection, optical waveguides, directional couplers, Kerr-lens mode-locking, optical power filter, optical fiber amplifier, multiplexers and demultiplexers *etc* [6].

There are three main mechanisms which are responsbile for the phenomena of self-focusing and defocusing of laser beams in dielectric, namely electrostriction, Kerr effect and thermal effect [7]. Out of these, thermal effects play a significant role in absorbing media. An intense light beam, with a radial distribution of intensity, causes a radial gradient of temperature and hence of dielectric constant. This leads to focusing or defocusing of the beam depending on the nature of variation of the beam with regard to the medium parameters. A lot of investigation has been carried out on the thermal self-focusing of laser beams in semiconductors, dielectrics and plasmas, both theoretically [8] and experimentally [9–13]. Steady state and non-steady state thermal self-focusing of an intense laser beam in

* Corresponding Author

dielectric medium has also been studied by Ghatak and Sharma [14].

Optical fibers with parabolically graded refractive index profile (also known as SELFOC fibers) are nowadays used extensively in optical communication systems due to their large information carrying capacity and low dispersion [15]. The ray paths are usually sinusoidal in such fibers [16]. The optical characteristics of a fiber waveguide depend largely on its refractive index profile [17,18]. The fiber geometry parameters are responsible for limiting the information carrying capacity of the beam due to their significant effect on dispersion and attenuation. There is a clearly pronounced tendency towards shortening of laser pulses. Pulses of picosecond duration are nowadays common. It is, therefore, necessary to take into consideration the nonstationary processes and the relaxation effects.

In the present analysis, we study the effect of thermal nonlinearity on the fiber parameters and propagation characteristics of the laser beam. Following the approach adopted by Akhmanov *et al* [1] and Sodha *et al* [2] and employing the paraxial ray approximation for self-focusing, a mathematical modelling has been developed using numerical computation. The present theory may find its use in manufacturing nonlinear optical passive device such as a new kind of thermal optical directional coupler.

Our analysis has been discussed in three sections. First, we discuss steady state thermal effects. In the second section, time dependent thermal effects have been discussed at length. In the last, a comparative analysis of the steady state and time-dependent thermal self-focusing is done and the results achieved thereby are compared with the available experimental data.

2. Steady state thermal effects

2.1 Effective dielectric constant of the medium .

Let us consider a cylindrical dielectric optical fiber waveguide having core radius a. The core is surrounded by a cladding region. Here, we assume that there is a geometrically built-in parabolic gradient of dielectric constant inside the core. For such a medium, the dielectric constant distribution is given by [15]

$$\begin{aligned} \varepsilon_0 &= \varepsilon_1\left[1-2\Delta(r/a)^2\right] \quad \text{for } 0 < r \le a, \\ &= \varepsilon_1\left[1-2\Delta\right] \quad \text{for } r > a, \end{aligned} \tag{1}$$

where $\Delta = \dfrac{\varepsilon_1 - \varepsilon_2}{2\varepsilon_1}$ a dimensionless parameter,

ε_1 = dielectric constant of the core at its axis,

ε_2 = dielectric constant of the cladding.

Let us also consider that the medium has finite (non-zero) thermal conductivity. In the presence of radial conduction of heat, the dielectric constant of the core can be written as

$$\varepsilon = \varepsilon_0 + \left[T(r) - T(0)\right]\frac{d\varepsilon_1}{dT}. \tag{2}$$

Here, $\dfrac{d\varepsilon}{dT}$ is the thermal gradient of the dielectric constant of the fiber core and $[T(r) - T(0)]$ is the temperature variation near the axis.

A cylindrically symmetric Gaussian laser beam is incident normally on the plane surface of the fiber (whose axis is coincident with the z axis) at $z = 0$. The radial intensity distribution of the beam is given by [8]

$$A_0^2(r,z) = \frac{E_0^2}{f^2(z)}\exp \quad \frac{}{r_0^2 f^2(z)} + \alpha z \tag{3}$$

Here, r_0 is the initial width of the laser beam and α is the intensity absorption coefficient of the beam which accounts for the losses and attenuation of the beam. Since the fiber is made up of an absorbing material, the incident power P_i attenuates as the beam propagates along the z direction

$$P = P_i \exp(-\alpha z). \tag{4}$$

At any radial point r, the fraction of available axial power will therefore be given by

$$P(r,z) = P\left| 1 - \exp\right| - \frac{}{r_0^2 f^2} \tag{5}$$

The power absorbed from the beam by the fiber core will be conducted across its curved surface.

In steady state, the equation for the conduction of temperature $T(r)$ along the curved surface over a section Δz will be given by (considering only the magnitude)

$$2\pi r \Delta z K \frac{\partial T(r)}{\partial r} \quad \frac{\partial P(r,z)}{\partial z} \Delta z \tag{6}$$

Here, K is the thermal conductivity of the fiber medium in W m^{-1} °C^{-1}. We have used the Taylor series expansion for the paraxial ($r << a$) distribution of temperature, and, making use of eq. (5), we obtain the variation in temperature

$$\begin{aligned} T(r) - T(0) &\approx r\left(\frac{\partial T}{\partial r} \quad\quad 2\, r^2\left(\frac{\partial^2 T}{\partial r^2}\right)_{r=0}\right. \\ &= -\frac{r^2}{2}\left(\frac{\alpha P_i}{2\pi K r_0^2 f^2}\right)\exp(-\alpha z). \end{aligned} \tag{7}$$

Eq. (2) takes the form

$$\varepsilon = \varepsilon_0 - \frac{\alpha P_i \exp(-\alpha z)}{2\pi K r_0^2 f^2}\frac{d\varepsilon_1}{dT} r^2 \tag{8}$$

Substituting eq. (1) in eq. (8), and rearranging the terms, one obtains the following equation :

$$\begin{aligned} \varepsilon &= \varepsilon_1\left[1 - 2\delta\left(\frac{r}{a}\right)^2\right] \quad \text{for } 0 < r \le a, \\ &= \varepsilon_1\left[1 - 2\delta\right] \quad \text{for } r > a, \end{aligned} \tag{9}$$

where $$\delta = \Delta + \frac{\alpha P_i \exp(-\alpha z)}{8\pi K \varepsilon_1}\frac{d\varepsilon_1}{dT}\left(-\frac{a^2}{r_0^2 f^2}\right) \qquad (10)$$

From eq. (10), it is obvious that the parameter δ, apart from depending on the geometry, also depends on the dielectric thermal gradient. Since we are following the paraxial ray approach, in which only the rays near to the axis are considered, the propagaton characteristics of the beam in the cladding region can be overlooked. Therefore, our discussion will be confined to the near-axis region ($r << a$) of the core only.

2.2 Wave equation for self-focusing :

The wave equation for a non-magnetic and non-(electrically) conducting medium, obtained by solving Maxwell's equations can be written as [7,8]

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} + \nabla\left(E.\frac{\nabla\varepsilon}{\varepsilon}\right) = 0\,, \qquad (11)$$

where c is the velocity of electromagnetic beam. Now, following Akhmanov *et al* [1] and Sodha *et al* [2] and applying the WKB approximation for paraxial rays, one gets

$$\frac{1}{k^2}\nabla^2(\ln\varepsilon) << 1\,. \qquad (12)$$

Eq. (12) gives the correct approximation here because, for paraxial rays, the higher order terms will be small and negligible and the nonlinearity only acts as perturbation. So, we can write eq. (11) as

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0\,. \qquad (13)$$

Now, we consider a plane wave solution of the type

$$E = A(z,r)\exp[i(\omega t - kz)] \qquad (14a)$$

with $$A(z,r) = A_0(z,r)\exp[-ikS(z,r)]\,, \qquad (14b)$$

where k = propagation constant of the beam,

ω = angular frequency of the signal,

$A(z, r)$ = complex envelope of the field,

$A_0(z, r)$ = amplitude of the envelope

and $S(z, r)$ = plane wave eikonal.

Using cylindrical coordinates and the plane wave solution, we obtain the wave equation as

$$2ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - 2k^2\delta\left(\frac{r}{a}\right)^2 A\,. \qquad (15)$$

On separating the real and imaginary parts of eq. (15), one can obtain the following two equations

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) - 2\delta\left(\frac{r}{a}\right)^2 \qquad (16a)$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0 \qquad (16b)$$

Let the general solution to be of the Gaussian form, eq. (3), with

$$S(z,r) = \frac{r^2}{2}\beta(z) + \phi(z)\,, \qquad (17)$$

also $$\beta(z) = \frac{1}{f}\frac{df}{dz}\,. \qquad (18)$$

Here, $f = f(z)$ is the dimensionless beam-width parameter and $r_0 f$ is the width of the beam.

Using eqs. (17), (18) and (14b) in eq. (16a) and collecting powers of r^2 on both sides of the resulting eq. [1,2], we get the second order beam-width equation

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2 r_0^4 f^3} - 2\frac{f}{r_0^2}\delta \qquad (19)$$

where δ is given by eq. (10).

2.3. Condition for uniform waveguide propagation and critical power :

When the diffraction divergence of the laser beam is exactly balanced by the focusing effect in the medium, the beam propagates in a self-trapped waveguide mode without convergence or divergence. Thus, applying the initial condition, that a $z = 0, f = 1$ and equating the R.H.S. of eq. (19) equal to zero, one can obtain [7,8]

$$\frac{\omega r_0}{c} = \left[2\varepsilon_1\left\{\Delta + \frac{\alpha(P_i)_{cr}}{8\pi K\varepsilon_1}\frac{d\varepsilon_1}{dT}\left(\frac{a}{r_0}\right)^2\right\}\right]^{-\frac{1}{2}} \qquad (20)$$

In the above equation, $(P_i)_{cr}$ is the critical power corresponding to the above condition. The critical power can be written as

$$(P_i)_{cr} = \frac{4\pi K\varepsilon_1}{\alpha(kr_0)^2(d\varepsilon_1/dT)}\left[1 - 2\Delta\left(\frac{r_0}{a}\right)^2\right] \qquad (21)$$

3. Time dependent thermal effects

3.1. Solution for the effective dielectric constant :

In order to study the time dependent thermal effects in a parabolic index optical fiber, the variation of the dielectric constant can be assumed to be of the following form

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon_1}{dT}T(r,z,t)\,. \qquad (22)$$

Here, ε_0 is given by eq. (1), the thermal gradient of the dielectric constant is $d\varepsilon_1/dT$ and $T(r, z, t)$ is the temperature rise which depends on time in the present section. Because of the cylindrical symmetry of the core and the radial conduction of heat, we have neglected the azimuthul coordinates and the term $\partial^2 T/\partial z^2$ which is considerably small [9,14]. Therefore, the heat conduction equation becomes

$$\rho C_p\frac{\partial T}{\partial t} = K\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + Q(r,z,t)\,, \qquad (23)$$

where ρ is the density of the medium in kg/m^3, C_p is the specific heat in J kg^{-1} °C^{-1}, K is the thermal conductivity in W m^{-1} °C^{-1} and Q is the source term (W/m^3) due to heating of the medium by the laser beam.

In order to solve the wave eq. (13) for which the dielectric constant distribution is given by (22), one has to determine the temperature function $T(r, z, t)$. For this, we have solved eq. (23) subject to the initial boundary conditions by using the method of separation of variables [19]. Hence, one gets

$$T(r,z,t)=\frac{2}{a^2}\sum_i\frac{J_0(r\xi_i)}{\left[J_1(a\xi_i)\right]^2}\int_0^t Q'(\xi_i,t)\times\exp\left[-\frac{(t-t')r_0^2\xi_i^2}{\tau}\right]dt' \tag{24}$$

with $$Q'(\xi_i,t)=\int_0^a Q(r,z,t)rJ_0(r\xi_i)\,dr\,, \tag{25}$$

where in view of the above eq. (5), we can assume the source term as

$$Q(r,z,t)=\frac{\alpha P_l\exp\{(-(\alpha z+t/\tau))\}}{\pi r_0^2 f^2}\left[1-\exp\left(-\frac{r^2}{r_0^2 f^2}\right)\right] \tag{26}$$

Here, $J_0(r\xi_i)$ and $J_1(a\xi_i)$ are the Bessel functions of order zero and one respectively. The constants ξ_i are the positive roots of the equation $J_0(a\xi_i) = 0$ with i being a positive integer. Also, $\tau = r_0\ \rho\ C_p/(\alpha K)$ is the relaxation time (in seconds) for the thermal stationary state to set in. For the paraxial region, the formula for temperature rise takes the form

$$T(r,z,t)=\frac{\alpha P_l e^{-\alpha z}}{4\pi K}\frac{r^2}{r_0^2 f^2}\left[1+\exp\left(-\frac{r^2}{r_0^2 f^2}\right)-\exp\left\{-\frac{r^2}{r_0^2 f^2\left(1+t/(\tau f^2)\right)}\right\}\right]. \tag{27}$$

In derivation of the above equation, we have taken the initial boundary conditions as : initially, when $t = 0$ and $z = 0$, the temperature function is a function of r only, and at $r = 0$, $dT/dr = 0$. Plot of the temperature function with radial distance from the axis shows gradual decrease as one moves away from the axis in radially outward direction (see Figure 1). Substituting for ε_0 and $T(r, z, t)$ using eqs. (1) and (27) respectively and then simplifying, the eq. (22) for dielectric constant takes the following form for the core region

$$\varepsilon=\varepsilon_1\left[1-2\delta_t\left(\frac{r}{a}\right)^2\right] \tag{28}$$

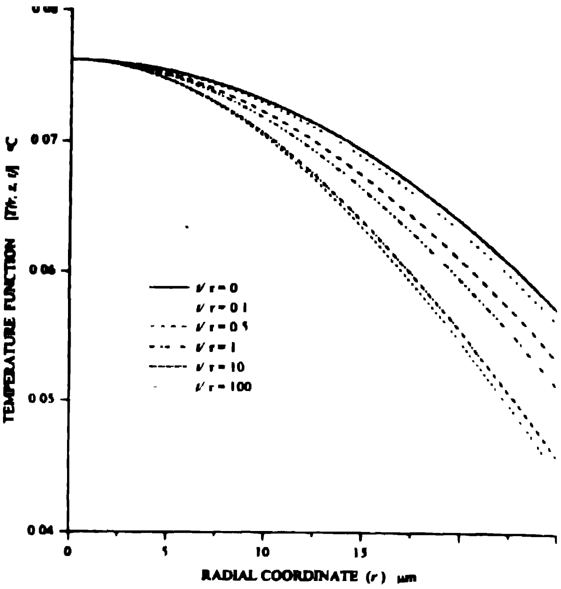

Figure 1. Variation of the temperature function $T(r, z, t)$ with the radial coordinate (r) in steady state and time dependent analysis Here, $r_0 = 25$ μm, $\alpha = 0.2$ cm^{-1}, $P_l = 10$ W, $K = 2.3\times10^{-3}$ Wm^{-1}°C^{-1}

where δ_t is given by

$$\delta_t=\Delta+\frac{\alpha P_l e^{-\alpha z}}{8\pi K\varepsilon_1}\frac{d\varepsilon_1}{dT}\frac{a^2}{r_0^2 f^2}\left[1+\exp\left(-\frac{r^2}{r_0^2 f^2}\right)-\exp\left\{-\frac{r^2}{r_0^2\left(f^2+t/\tau\right)}\right\}\right] \tag{29}$$

The graph of radial variation of ε is shown in Figure 2. It is worthwhile to mention here that if we substitute $t = 0$

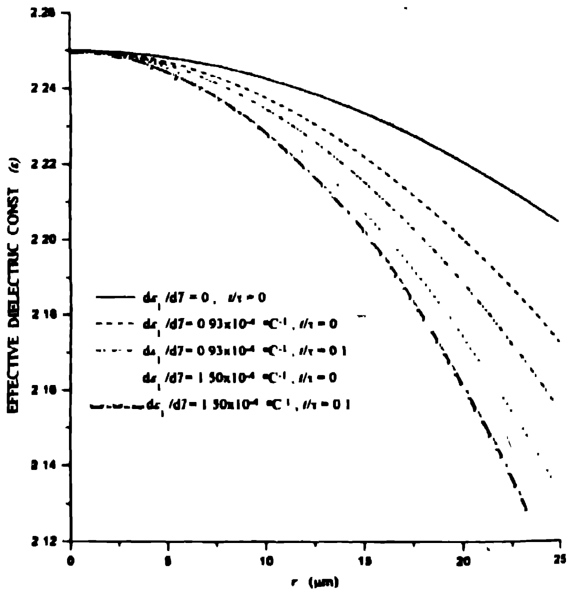

Figure 2. The effective dielectric constant (ε) variation with the radial coordinate (r) for different values of the dielectric thermal gradient ($d\varepsilon_l/dT$) and the normalised time (t/τ). Here, $a = 25$ μm $= r_0$, $\alpha = 0.2$ cm^{-1}, $P_l = 10$ W, $K = 2.3\times10^{-3}$ Wm^{-1}°C^{-1}, $\Delta = 0.01$.

in eq. (28), it reduces to the eq. (9) related to the steady state case.

3.2 Second order beamwidth equation :

We follow the method adopted in Section 2 to solve the wave equation. Employing the WKB approximation and making use of the plane wave eikonal, one gets

$$2ik\frac{\partial A}{\partial z}=\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}-\frac{2k^2 A}{\varepsilon_1}\delta_t\left(\frac{r}{a}\right)^2. \tag{30}$$

Substituting for $A(r, z)$ from eq. (14b), and then separating the real and imaginary parts, one obatins the following set of real and imaginary parts of the above equation

$$2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{k^2 A_0}\times\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)-2\delta_t\left(\frac{r}{a}\right)^2 \tag{31a}$$

and
$$\frac{\partial A_0^2}{\partial z}+\frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r}+A_0^2\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)=0. \tag{31b}$$

By making use of eqs. (17) and (18) and then collecting the powers of r^2, we obtain from the above eq. (30), which is again the second order nonlinear equation for the dimensionless beamwidth parameter

$$\frac{d^2 f}{dz^2}=\frac{1}{k^2 r_0^4 f^3}-2\frac{f}{r_0^2}\delta_t. \tag{32}$$

In the above eq. (32), the parameter δ_t is given by eq. (29).

3.3 Critical power for the time-dependent case :

Whenever the divergence caused due to diffraction exactly equals to the collimating (focusing) term, self-trapping of the beam occurs. Hence, in the eq. (32), let us apply the boundary condition as stated in Section 3.1 and following the approach of Sodha and coworkers [7,8], we have

$$\frac{\omega r_0}{c}=\left[2\varepsilon_1\left\{\Delta+\frac{\alpha(P_t)_{cr}}{8\pi K\varepsilon_t}\frac{d\varepsilon_1}{dT}\left(\frac{a}{r_0}\right)^2\right.\right.\times\left.\left.\left[1+\exp\left(-\frac{r^2}{r_0^2}\right)-\exp\left(-\frac{r^2}{r_0^2(1+t/\tau)}\right)\right]\right\}\right]^{-\frac{1}{2}}. \tag{33}$$

In the above equation, $(P_t)_{cr}$ is the critical power corresponding to the above condition. Solving the equation, the critical power in the presence of time-dependent thermal nonlinearity may be written as

$$(P_t)_{cr}=\frac{4\pi K\varepsilon_1}{\alpha(kr_0)^2(d\varepsilon_1/dT)}\left[1-2\Delta\left(\frac{r_0}{a}\right)^2\right]\times\left[1+\exp\left(\frac{r^2}{r_0^2 f^2}\right)-\exp\left(\frac{r^2}{r_0^2(1+t/\tau)}\right)\right]^{-1}. \tag{34}$$

4. Results and discussion

The foregoing analysis of laser beam propagation through the optical fiber may be useful in obtaining many useful results. Some of the salient results are discussed in this section. In the end, a comparison with the experimental results is done and the importance of the theory is discussed.

In Figure 1, the radial profiles of temperature function are displayed. As one moves outwards in the radial direction, the temperature rise due to heating of the medium diminishes. The rise in temperature acts as perturbation on the geometrically built-in gradient of the dielectric constant in those dielectrics for which $d\varepsilon_1/dT > 0$ as in the case of silica fibers. This causes an enhancement in the focusing property of the fiber. A comparison of the solid curve, drawn for the steady state case, with the dotted curves (drawn for the time factors $t/\tau = 0.1$ and $t/\tau = 1$ respectively) shows that the effect of temperature rise diminishes as the time elapses. As the duration of pulse exceeds the relaxation time, the rise in temperature saturates.

The solid curve in Figure 2 shows a gradual decrease of the dielectric constant in the radial direction due to parabolic geometry of the optical fiber. If the core material has also an additive thermal nonlinearity, then the fall in the dielectric constant in the outward radial direction may be noticed steeper as the dotted curves show. The time dependent analysis predicts lower values of the dielectric constant for the same radial distance. This behaviour may affect the propagation characteristics of fiber and may cause enhancement in the focusing or de-focusing of laser beam.

From the eq. (10) and (29), it may be noticed that the parameters δ and δ_t stand for a change in the geometrical factor Δ. As the normalised time factor approaches infinity, δ_t saturates to a constant value (see Figure 3). The attenuation constant α also affects δ_t as the graphs show. From Figure 3, one may infer that, for the laser pulses which are of very short duration, *i.e.* $t \ll \tau$, transient region behaviour will play an important role in the propagation characteristics. When the duration of pulse considerably exceeds the relaxation time τ of the nonlinear medium, the quasistatic region parameters will govern the propagation behaviour. For the thermal nonlinearity, the relaxation time is the time of thermo-diffusion and its typical values are 0.1 to 1 second.

The dimensionless beamwidth parameter f is an important factor in the discussion of self-focusing/defocusing of laser beam. In many studies [20–28], this has been solved specially for the plasma medium. We have solved for the same using eqs. (19) and (32) employing the second order Runge-Kutta method and the condition of self-trapped waveguide [3,4,8].

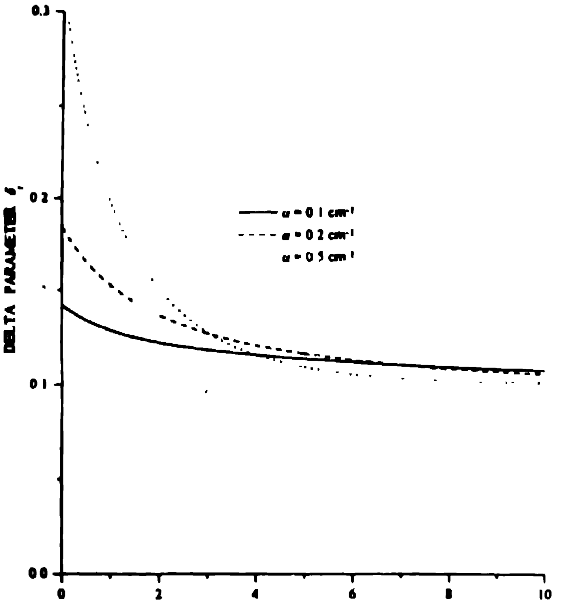

Figure 3. Variation of the delta parameter (δ) with normalised time (t/τ) with different values of attenuation constant (α). We have, P_i = 10 W, $d\varepsilon_l/dT$ = 0.93 × 10^{-4} °C^{-1}, ε_l = 2.25, $a = r_0$ = 25 μm, K = 2.3 × 10^{-3} Wm^{-1}°C^{-1}, Δ = 0.01

In the fiber, the causes of beam attenuation are mostly fiber imperfections, microbending losses and dispersion. The dotted plot in Figure 4 shows that the laser beam goes unattenuated in the absence of attenuation. The solid curve displays attenuation of the beam when thermal nonlinearity

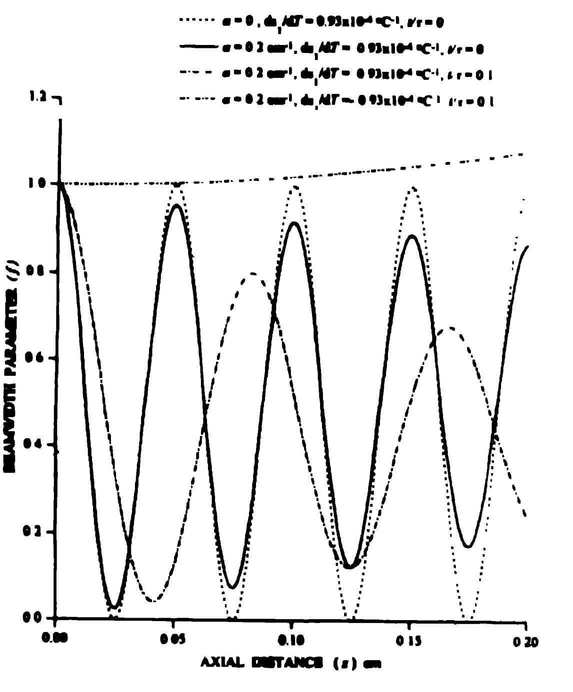

Figure 4. Variation of the beamwidth parameter (f) with axial distance (z) for different values of attenuation constant (α) and dielectric thermal gradient ($d\varepsilon_l/dT$), with P_i = 10 W, ε_l = 2.25, $a = r_0$ = 25 μm, K = 2.3 × 10^{-3} Wm^{-1}°C^{-1}, Δ = 0.01.

is considered. Temporal considerations show a shift in the focusing points along the axis and also a more attenuated beam. If a material with negative thermal gradient is taken, a defocusing effect may be observed.

In Figure 5, the variation of $\omega r_0/c$ with the input critical power $(P_i)_{cr}$, using eqs. (20) and (33), is shown. In earlier

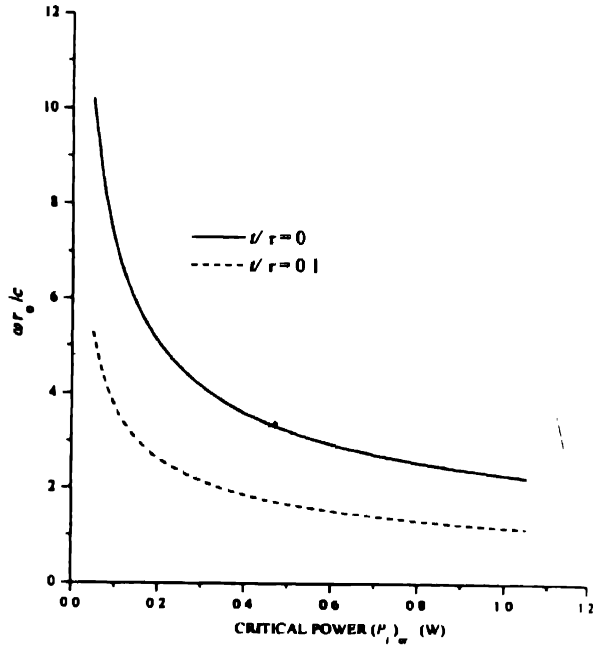

Figure 5. Plots of the quantity ($\omega r_0/c$) with the critical input power $(P_i)_{cr}$ for different values of the time factor (t/τ). In these graphs, $d\varepsilon_l/dT$ = 0.93 × 10^{-4} °C^{-1}, ε_l = 2.25, $a = r_0$ = 25 μm, K = 2.3 × 10^{-3} Wm^{-1}°C^{-1} and Δ = 0.01

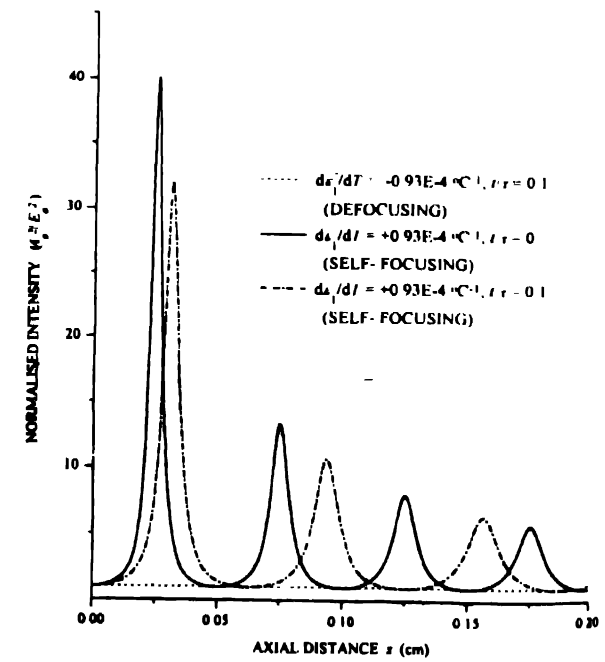

Figure 6. Normalised axial intensity variation with the axial distance (z) in stationary and time dependent case for different values of thermal gradient ($d\varepsilon_l/dT$). Here, α = 0.2 cm^{-1} and $r_0 = a$ = 25 μm

studies [2,21,22,24], two input critical powers in plasma medium have been reported. As the present discussion shows, for a given radius there is a single value of $(P_i)_{cr}$. But if the time of observations are different, one may get different values of critical power for self-focusing at the same radial distance. At r_0 = 4 μm, when t/τ = 0, $(P_i)_{cr}$ = 0.3 W and when t/τ =0.1, $(P_i)_{cr}$ = 0.1 W.

A shift in the intensity peaks may be noticed as shown in Figure 6. In case of focusing, a lowering of the maximum intensity takes place due to attenuation. The first intensity peak for t/τ = 0 is at (0.025, 40), whereas for t/τ = 0.1, it is at (0.03, 32). In case of de-focusing, the plot shows absence of peaks and slight attenuation.

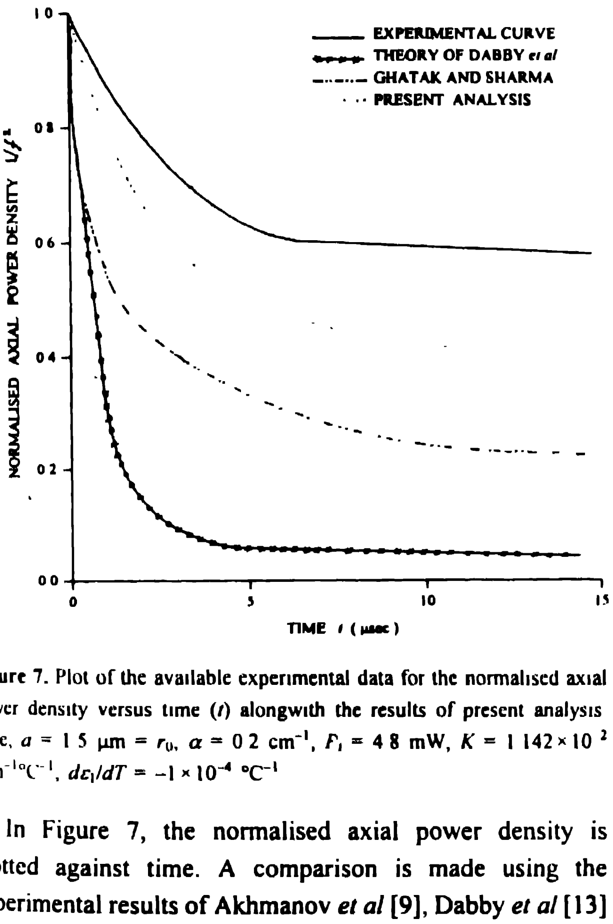

Figure 7. Plot of the available experimental data for the normalised axial power density versus time (t) alongwith the results of present analysis Here, a = 1.5 μm = r_0, α = 0.2 cm^{-1}, P_i = 4.8 mW, K = 1.142 × 10^{2} Wm^{-1}°C^{-1}, $d\varepsilon_1/dT$ = -1×10^{-4} °C^{-1}

In Figure 7, the normalised axial power density is plotted against time. A comparison is made using the experimental results of Akhmanov *et al* [9], Dabby *et al* [13] for a dye sample using a He-Ne laser with output power of 4.8 mW at 6328 Å in TEM_{00} mode and Ghatak and Sharma [14]. This figure shows that the results of the present modelling of the problem are in good agreement with the experimental results as compared with those of [13] and [14].

Figure 8 shows the variation of the delta parameter with the log of input power. It may be seen that this parameter shows an increase with increase in input power. Results indicate that the pulses of different time duration and power will have different propagation characteristics for the same type of fiber. This information might be useful for designing different fiber devices.

Thermal self-focusing parameters in different materials are shown in Table 1. Our results indicate that the intensity gain for time dependent case is slightly lower than the stationary state case. Nonlinear focal length z_f for silica fiber is very small. The smaller value of nonlinear focal length may be ascribed to the fact that focusing properties of the medium increase due to the thermal nonlinearity. The beam

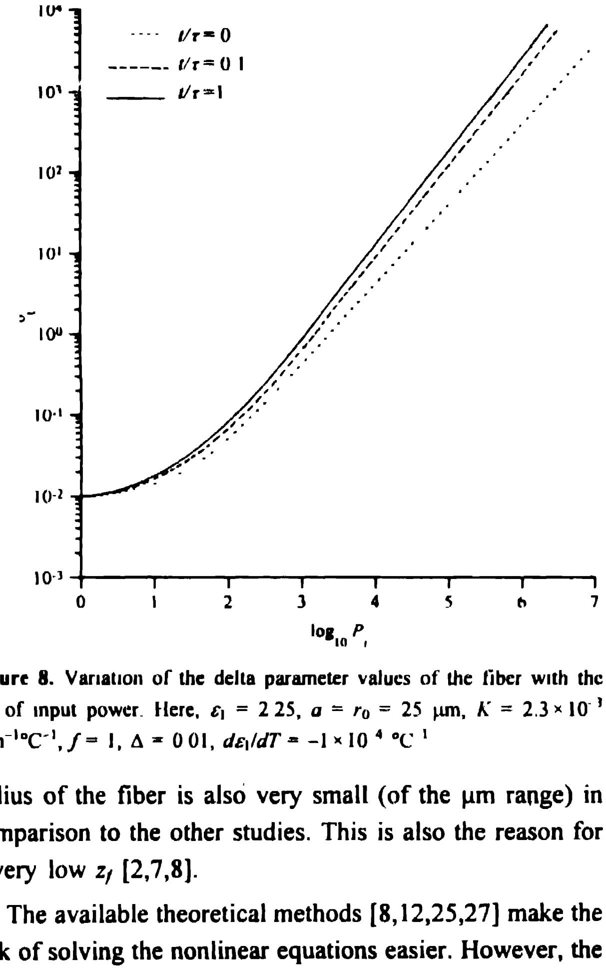

Figure 8. Variation of the delta parameter values of the fiber with the log of input power. Here, ε_1 = 2.25, a = r_0 = 25 μm, K = 2.3 × 10^{-3} Wm^{-1}°C^{-1}, f = 1, Δ = 0.01, $d\varepsilon_1/dT$ = -1×10^{-4} °C^{-1}

radius of the fiber is also very small (of the μm range) in comparison to the other studies. This is also the reason for a very low z_f [2,7,8].

The available theoretical methods [8,12,25,27] make the task of solving the nonlinear equations easier. However, the non-paraxial region may be explored by using other [25, 28] methods. In recent years, there has been an interest in the techniques in which light is controlled or manipulated by light [29]. The results presented here indicate that the propagation characteristics of Gaussian laser beam in the parabolic index optical fiber are very sensitive to the thermal nonlinearity. By properly selecting fiber parameters, critical power, attenuation constant, conductivity *etc.*, the intensity of light can be controlled in a predetermined and predictable manner.

Table 1. Thermal self-focusing parameters in different materials

Sample material	Laser beam input power P_i (W)	Beam radius r_0 (mm)	Sample length l (cm)	Absorption coeff. α (cm^{-1})	Nonlinear focal length z_1 (cm)	Intensity gain: Stationary case	Intensity gain: Time-dependent case
$LiNbO_3$* + Nd	0.6	0.35	0.4	9	6	–	–
Glass** TR-105	0.9	0.35	12	0.1	10	–	–
Lead@ glass	8	0.8	15 25 35	20	30	–	1
Glass'	–	–	–	–	10	–	–
Crown++ glass 5% Nd	5	1.2	30	0.04 0.5	30	–	200
Silica# glass fiber	10	25 (μm)	0.2	0.2	0.025	40	32

*Akhmanov *et al* [1], **Akhmanov *et al* [9], @Dabby *et al* [13], 'Dobby and Schmidt [30], ++Carman *et al* [31], #Present analysis.

Acknowledgement

Authors are thankful to Prof. R S Mina, Principal, Government College, Ajmer for useful discussion and providing facilities for the research work.

References

[1] S A Akhmanov, A P Sukhorukov and R V Khokhlov *Sov Phys Usp* **10** 609 (1968)

[2] M S Sodha, A K Ghatak and V K Tripathi *Progress in Optics* (ed.) E Wolf (Amsterdam North Holland) Vol. **XIII** p169 (1976)

[3] R Y Chiao, E Garmire and C H Towens *Phys Rev Lett* **13** 479 (1964)

[4] E Garmire, R Y Chiao and C H Towens *Phys Rev Lett* **16** 347 (1966)

[5] A Hasegawa and F Tappert *Appl Phys Lett* **23** 3 (1973)

[6] M M Fejer *Phys Today* **May 25** 43 (1994)

[7] M S Sodha *J Phys Edu* (India) **12** 13 (1973)

[8] M S Sodha, A K Ghatak and V K Tripathi *Self-focusing of Laser Beams in Dielectrics, Plasmas and Semiconductors* (New Delhi Tata McGraw-Hill) p22 (1974)

[9] S A Akhmanov, D P Krindach, A V Migulin, A P Sukhorukov and R V Khokhlov *IEEE J Quant Electron* **QE-4** 568 (1969)

[10] Yu P Raizer *Sov Phys JETP Lett* **4** 286 (1966)

[11] A G Litvak *Sov JETP Lett* **4** 341 (1966)

[12] Yu P Raizer *Sov Phys JETP Lett* **52** 470 (1967)

[13] F W Dabby, R W Boyko, C V Shank and J R Whinnery *IEEE J. Quant Electron* **5** 516 (1969)

[14] A K Ghatak and S K Sharma *Appl Phys Lett.* **22** 141 (1973)

[15] A K Ghatak and K Thyagarajan *Graded Index Optical Waveguides. A Review (Progress in Optics)* (ed.) E Wolf (Amsterdam North Holland) Vol. **XVIII** p1 (1980)

[16] A K Ghatak and K Thyagarajan *Optical Electronics* (Cambridge Cambridge University Press) 371 (1989)

[17] A K Ghatak and K Thyagarajan *J Opt Soc Am.* **65** 169 (1975)

[18] Ajit Kumar *Opt Comm.* **84** 846 (1991)

[19] Ian Snedden *Elements of Partial Differential Equations* (Int Student Edn) (1964)

[20] S Medhekar, M S Sodha and S Konar *Opt Lett* **21, 4** 305 (1996)

[21] R K Khanna, K Baheti and A K Nagar *Proceedings of Plasma-94 Symposium* (Indian Institute of Science, Guwahati, India) (1994)

[22] R K Khanna, K Baheti and A K Nagar *Proceedings of Plasma-97 Symposium* (Physical Research Lab., Ahmedabad, India) (1997)

[23] R K Khanna and K Baheti *Indian J Phys.* **73B** 73 (1999)

[24] K P Maheswari, S Konar and M S Sodha *J Plasma Phys* **48** 107 (1992)

[25] M S Sodha and D Subbarao *Contemporary Plasma Physics* (eds.) M S Sodha, D P Tiwari and D Subbarao (New Delhi Macmillan) p249 (1984)

[26] H Singh, D Subbarao and R Uma *High Performance Computing* (eds.) S Sahani, V K Prasanna and V P Bhatkar (New Delhi Tata McGraw-Hill) p169 (1996)

[27] D Subbarao, R Uma and A K Ghatak *Laser Beams* **Part I** 367 (1983)

[28] D Subbarao, R Uma and H Singh *Phys Plasmas* **5** 3440 (1998)

[29] Raj Kamal, K P Maheshwari and R L Sawhney (eds.) *Reviews in Contemporary Physics-Laser Optics* (New Delhi : Wiley Eastern) (1992)

[30] F W Dobby and R V Schmidth Symposium Division of Electron and Atomic Physics USA (1968)

[31] R L Carman, A Mooradian, P L Kelly and A Tufts *Appl Phys Letts.* **14** 136 (1969)